AP Calculus BC

WS 92 - Powers Series Review

1) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

Ratio Test

$\lim_{n\to\infty}\left[\frac{x^{n+1}}{(n+1)!}\cdot\frac{n!}{x^n}\right] = \frac{x}{n+1}$

$\lim_{n\to\infty}\frac{x}{n+1} = 0 < 1$

(D) Converges for all values of x

2) $\frac{e^x+e^{-x}}{2} = \frac{\left(1+x+\frac{x^2}{2!}+\cdots\right)+\left(1-x+\frac{x^2}{2!}-\cdots\right)}{2}$

$= \frac{2+2\left(\frac{x^2}{2!}\right)+2\left(\frac{x^4}{4!}\right)+\cdots}{2}$

$= 1+\frac{x^2}{2!}+\frac{x^4}{4!}$ (B)

3) $\sum_{n=0}^{\infty}\frac{(x+2)^n}{n+2} = S$

Ratio Test

$\lim_{n\to\infty}\left|\frac{(x+2)^{n+1}}{(n+3)}\cdot\frac{n+2}{(x+2)^n}\right| = |x+2|<1$

$-1<x+2<1$

$-3<x<-1$

ENDpoints

$x=-3$: $\sum_{n=0}^{\infty}\frac{(-1)^n}{n+2}$

AST

converges

$x=-1$: $\sum_{n=0}^{\infty}\frac{1}{n+2}$

LCT

$\sum\frac{1}{n}$ diverges

$\lim_{n\to\infty}\left[\frac{1}{n+2}\cdot n\right]=1$

diverges

$\boxed{S \text{ converges on } -3\le x<-1}$

4) $\sin x$

$f(x)=\sin x$

$f'(x)=\cos x$

$f''(x)=-\sin x$

$f'''(x)=-\cos x$

$f^{(4)}(x)=\sin x$

$f^{(4)}\left(\frac{\pi}{2}\right)=1$

coefficient: $\frac{1}{4!}$ (C)

5) $\sum_{n=0}^{\infty} 5x^n$

GST

$-1<x<1$

6) $f^{(6)}(x)=-\sin x$

$f^{(6)}\left(\frac{\pi}{2}\right)=-1$

coefficient: $\frac{-1}{6!}$ (D)

7) $$\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$x^3 = x^{2n+1}$$

$$2n+1 = 3 \qquad \text{coefficient}: (-1)^1 \cdot \frac{1}{3!}$$

$$2n = 2$$

$$n = 1 \qquad\qquad = \frac{-1}{6}$$

(E)

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!} = \overset{0}{1} - \overset{1}{x^2} + \overset{2}{\frac{x^4}{2!}} - \overset{3}{\frac{x^6}{3!}} + \overset{4}{\frac{x^8}{4!}} - \cdots$$

$$f'(x) = -2x + \frac{4x^3}{2!} - \frac{6x^5}{3!} + \frac{8x^7}{4!} - \cdots$$

$$f''(x) = -2 + \frac{12x^2}{2!} - \frac{30x^4}{3!} + \frac{56x^6}{4!} - \cdots$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n (2n)(2n-1)x^{2n-2}}{n!}$$ ← NOT REQUIRED FOR THIS PROBLEM, BUT INTERESTING TO THINK ABOUT

$$\boxed{f''(0) = -2}$$